A comparison of BWRs and PWRs with a “Simple Model”

A look at the paper
“A simple model of reactor cores for reactor neutrino flux calculations for the KamLAND experiment” by K. Nakajima, *et al*.

(presented by G. Horton-Smith)

Primary reference

A simple model of reactor cores for reactor neutrino flux calculations for the KamLAND experiment

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Abstract

KamLAND is a reactor neutrino oscillation experiment with a very long baseline. This experiment successfully measured oscillation phenomena of reactor antineutrinos coming mainly from 53 reactors in Japan. In order to extract the results, it is necessary to accurately calculate time-dependent antineutrino spectra from all the reactors. A simple model of reactor cores and code implementing it were developed for this purpose. This paper describes the model of the reactor cores used in the KamLAND reactor analysis.

Key words: Neutrino oscillation, Reactor antineutrino, Fission rate calculation, Nuclear fuel, KamLAND

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1 Introduction

The KamLAND (*Kamioka Liquid Scintillator Anti-Neutrino Detector*) experiment [1] is a reactor neutrino oscillation experiment with a very long baseline.

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- K. Nakajima, *et al.*, *"A simple model of reactor cores for reactor neutrino flux calculations..."*, physics/0607126, July 2006. (submitted to Elsevier Science)

Executive summary

- "The number of antineutrinos per unit energy generation changes as much as 10% as the components of nuclear fuel change during fuel burnup. It is therefore necessary to trace the burnup effect for each reactor.
- "For the case of KamLAND, it is practically impossible to calculate the burnup effect using a detailed simulation for each reactor.
- "This paper describes a simple reactor model with which to accurately calculate the $\bar{\nu}_e$ spectrum of each reactor using routinely recorded reactor operation parameters.
- "The parameters include the time-dependent thermal output, burnup, and the volume and enrichment of exchanged fuel."

Bottom line

- The simple model gives the same results as the detailed simulation to better than 1% for 5 BWRs and 1 PWR studied over several cycles.

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For reactor monitoring:

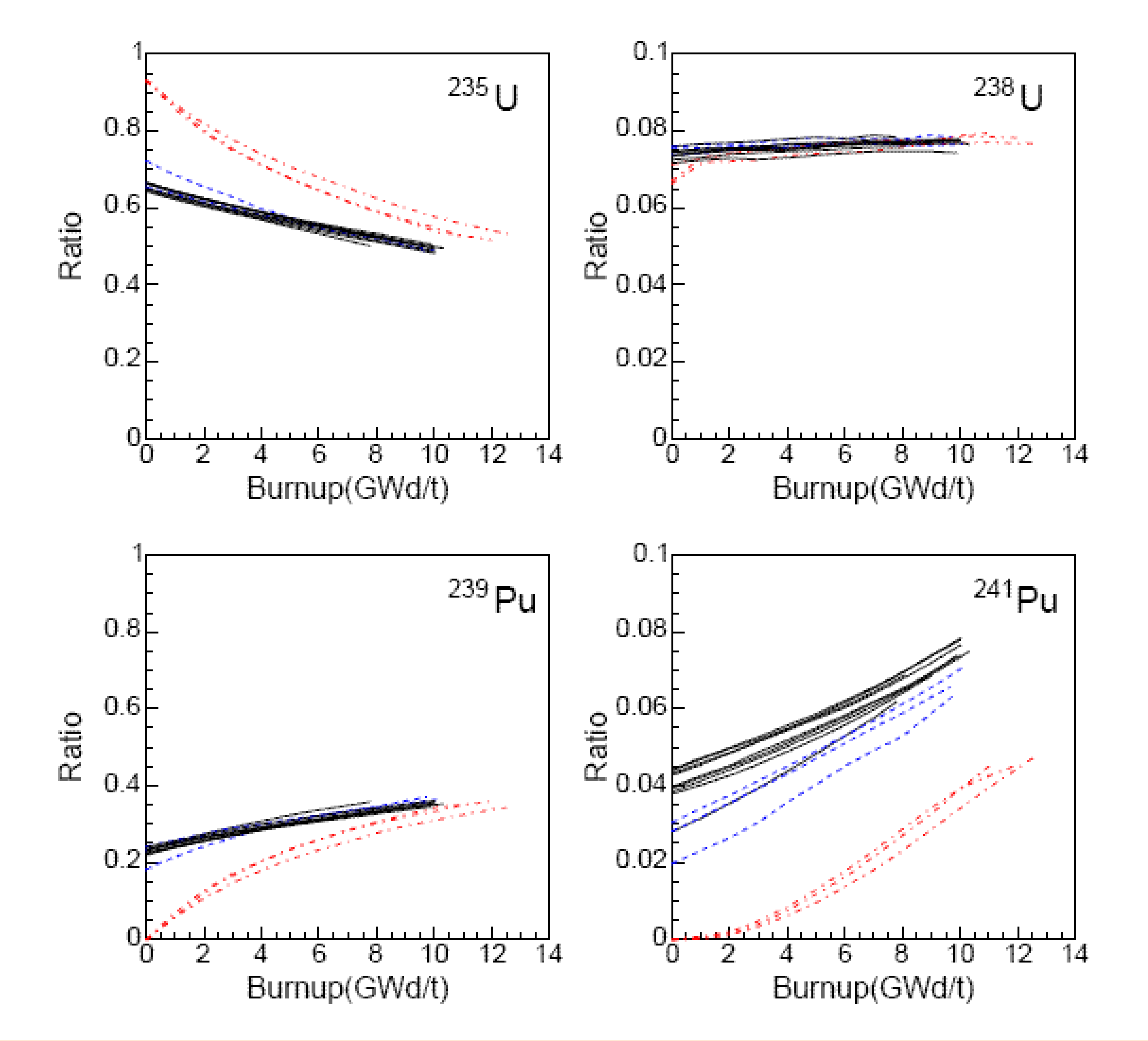
This provides a model for how to get the information needed to convert neutrino rate to thermal power or burnup effect to kg of Pu in the core without continuous extraordinary effort by the reactor operators.

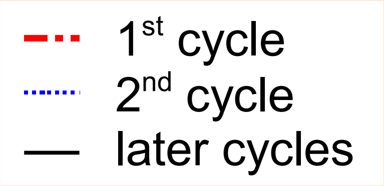
What were the detailed simulations?

- "Core Management System (CMS) codes from Studsvik of America, CASMO[6]/SIMULATE[7]."
- "A comparison [8] of the calculated isotopic concentrations provided by the CMS codes with experiments was carried out for the spent nuclear fuel discharged from a BWR type reactor core in Japan." Error less than 7% for the four main fissile isotopes, corresponding to <1% error in neutrino flux.

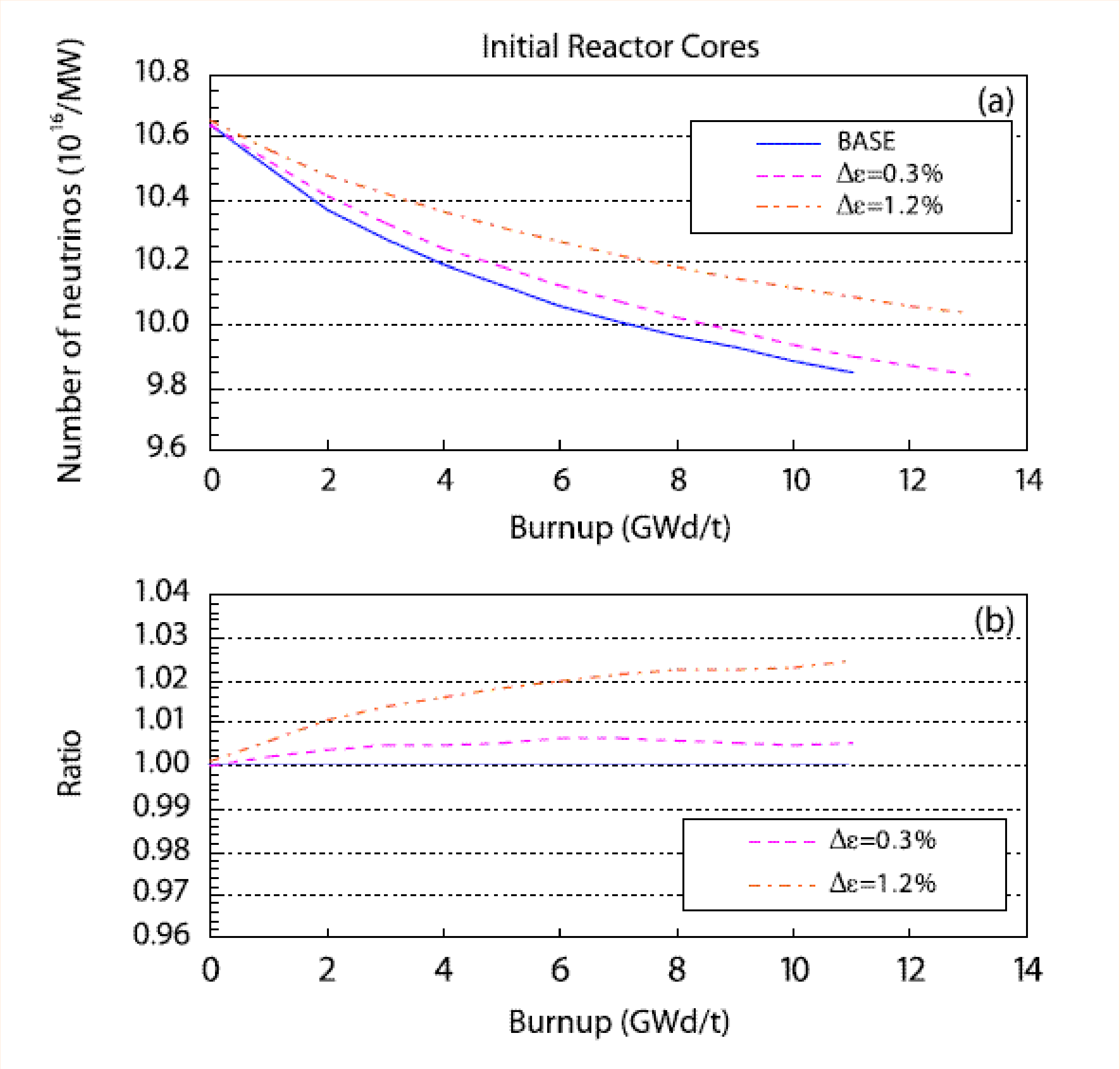
Core number	Core type	Cycle number	$\epsilon(\%)$	$V(\%)$
1	BWR	1	2.2	100
		2	3.0	40
		After 3	3.0	25
2	BWR	1	2.2	100
		2	3.4	25
		After 3	3.4	29
3	BWR	1	2.5	100
		2	3.4	28
		After 3	3.4	26
			3.4	28
			3.4	29
			3.7	26
4	BWR	After 3	3.7	26
5	BWR	After 3	3.4	23
			3.4	28
6	PWR	1	2.2	100
		2	3.5	33
		After 3	3.5	33

Relative fission yields from detailed simulation (Fig. 2 in Nakajima, *et al.*)





Neutrinos per MJ vs burnup from detailed simulation (Fig. 3)



3.4% ^{235}U
2.5% ^{235}U
2.2% ^{235}U

What is the simple model?

- For some reference enrichment ϵ_0, create a reference curve $N(\epsilon_0,b)$ for each quantity of interest N as a function of burnup b.
- Deal with varying enrichments in incoming fuel as a correction to b:

$$N(\epsilon_{.},b)=N(\epsilon_{.}+\Delta\epsilon,b+\beta(\epsilon_{.},b)\Delta\epsilon)$$

$$\text{or } N(\epsilon_{.}+\Delta\epsilon,b')=N(\epsilon_{.},b),\ \text{where } b'=b+\beta(\epsilon_{.},b)\Delta\epsilon$$

- To first order,

$$\beta(\epsilon_0,b)=-\frac{\partial N(\epsilon_0)}{\partial\epsilon}\Big/\frac{\partial N(b)}{\partial b}.$$

Details of the simple model

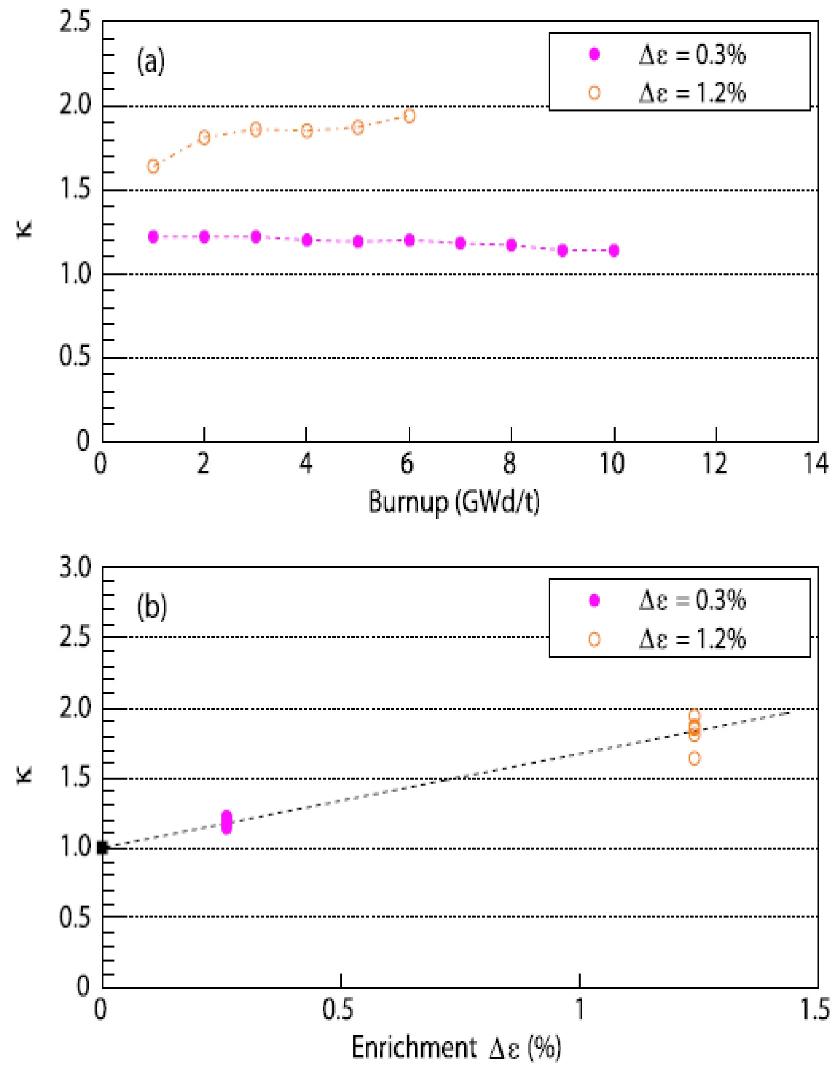
- The main effect of higher enrichment is a "stretching" of b, so define

$$\kappa = 1 + \frac{\beta(\epsilon_0, b)}{b}\Delta\epsilon.$$

- κ is nearly constant over the cycle, so

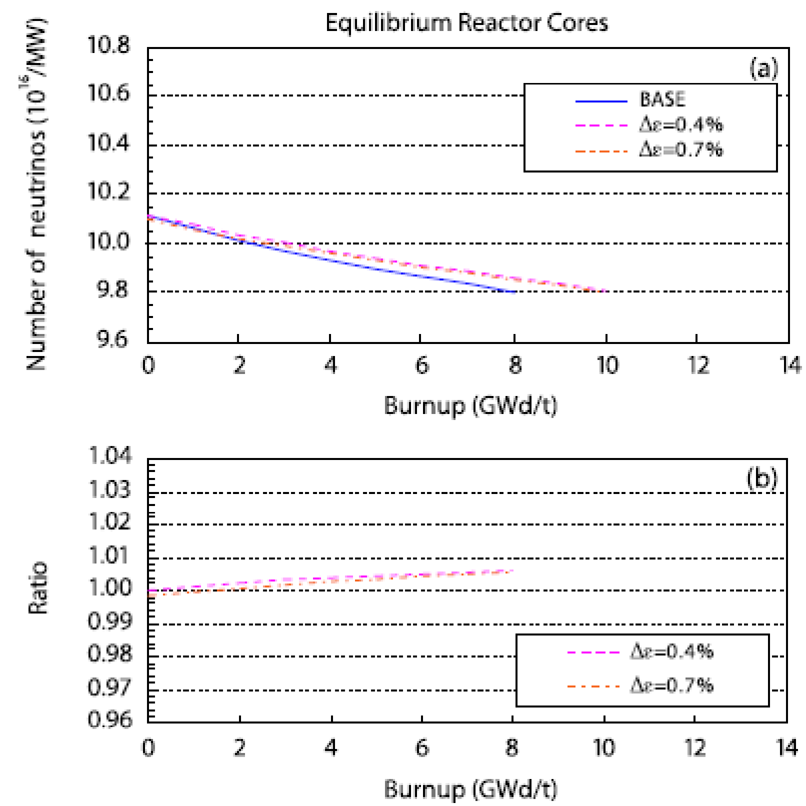
$$N\left(\epsilon_0 + \Delta\epsilon, b\right) = N\left(\epsilon_0, \frac{b}{\kappa}\right).$$

- κ is linear function of $\Delta\epsilon$.
  (Different slope for PWRs and BWRs.)



Further details of the simple model

- Calculate separate burnup corrections for initial and equilibrium cores.
- For the fuel fission ratios, there is an additional correction for averaged absolute burnup from previous cycles.
- Latter correction is <~5% except for ^{241}Pu in BWRs, where it is "<~30%".



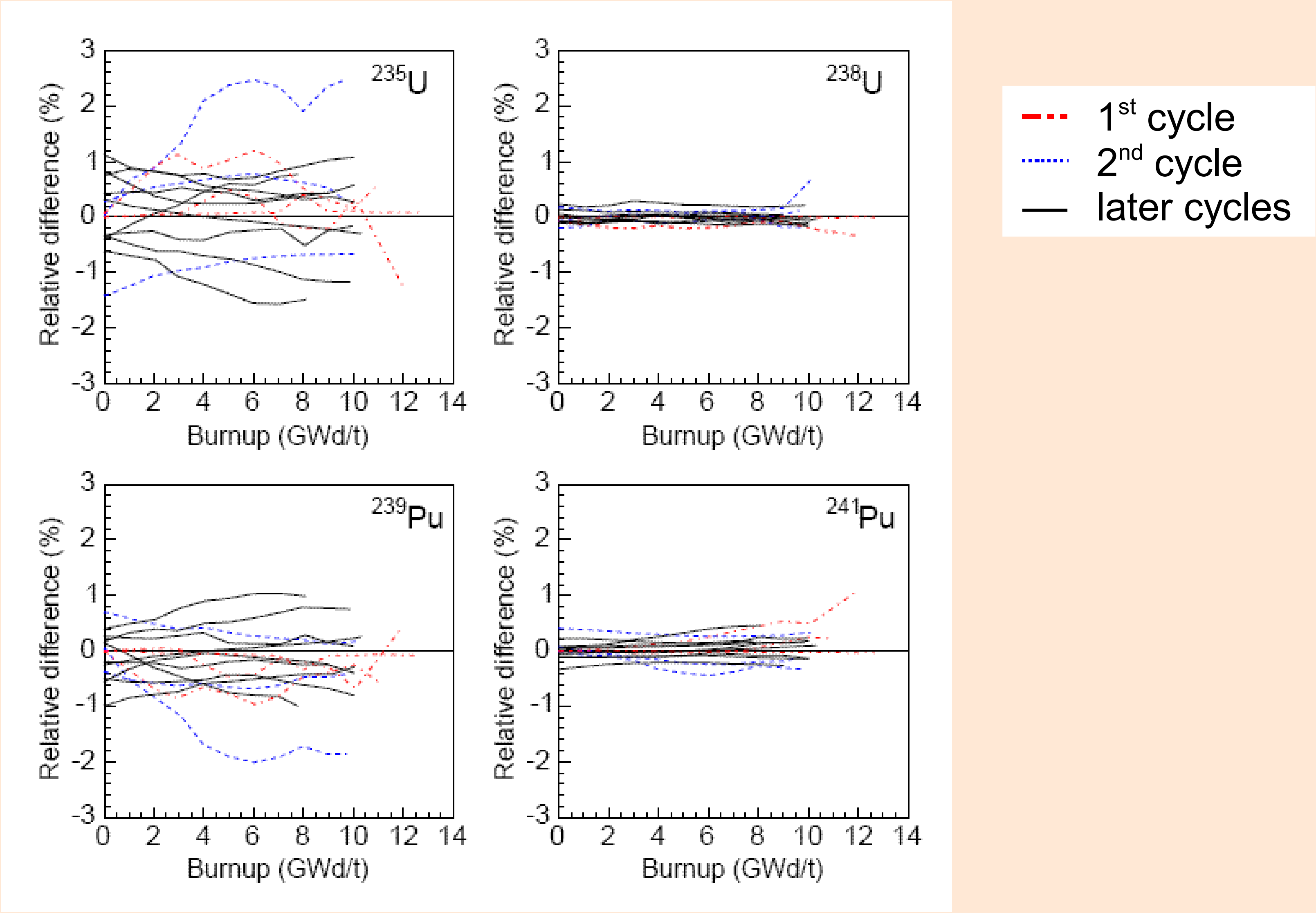
You want details, I'll give you details...

- Initial cores: $\kappa=1+0.65\Delta\epsilon(\%)$ and $\kappa=1+0.54\Delta\epsilon(\%)$ for the BWR and PWR cores, respectively.
- Equilibrium: $\kappa=1+0.35\Delta\epsilon(\%)$ and $\kappa=1+0.29\Delta\epsilon(\%)$ for the BWR and PWR cores, respectively.
- The correction term for fission ratios due to prior burnup history is

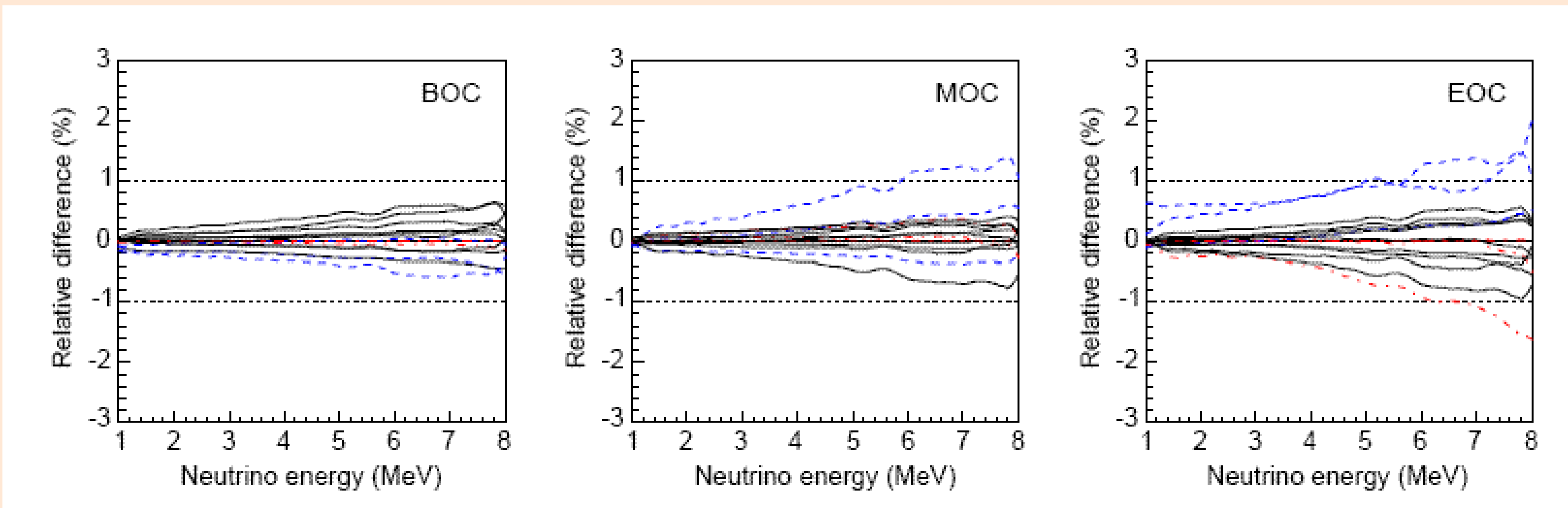
$$\Delta f^{\mathrm{isotope}}(b_{\mathrm{absolute}}) \equiv \eta^{\mathrm{isotope}} \left(\frac{b_{\mathrm{absolute}}}{b^{0}_{\mathrm{absolute}}} - 1 \right)$$

where η is a factor analogous to κ.

Difference between simple and detailed model: fissions (Fig. 6)



Difference between simple and detailed model: $\bar{\nu}$ spectra (Fig. 7)



Nakajima *et al.*'s summary

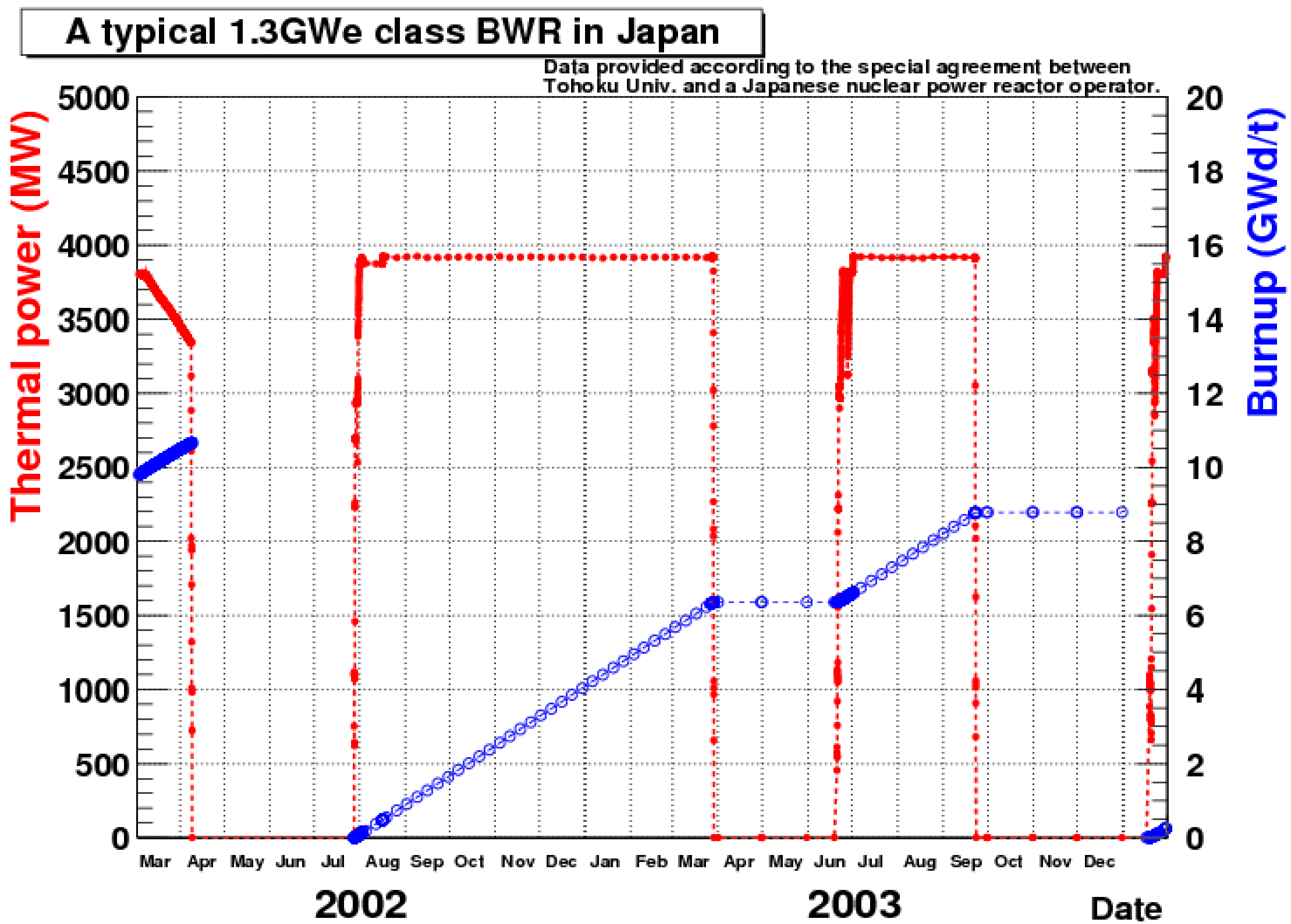
- "To calculate $\bar{\nu}_e$ flux, reactor cores can be successfully modeled using only a few reactor operation parameters.
- "The results of our simplified reactor model agree with detailed reactor core simulations within 1% for different reactor types and burnup. This error is taken into account in the KamLAND reactor neutrino analysis.
- "The simplified model may be applicable to future long-baseline reactor neutrino experiments which make use of several reactors."

Implications for neutrino-based reactor monitoring

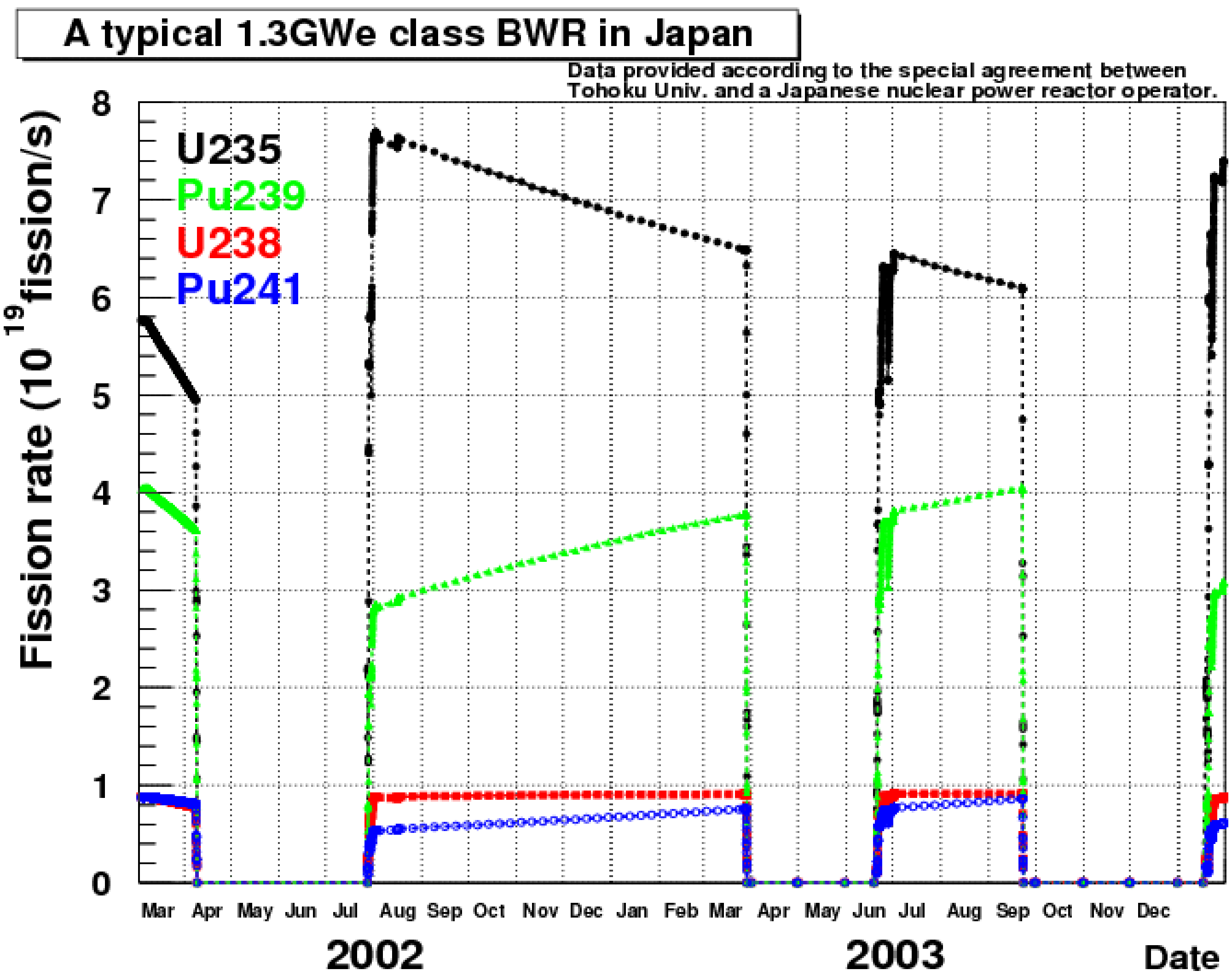
(None of the following is in the “Simple Model” paper, so blame me, not any of the “Simple Model” authors.)

Additional references used here

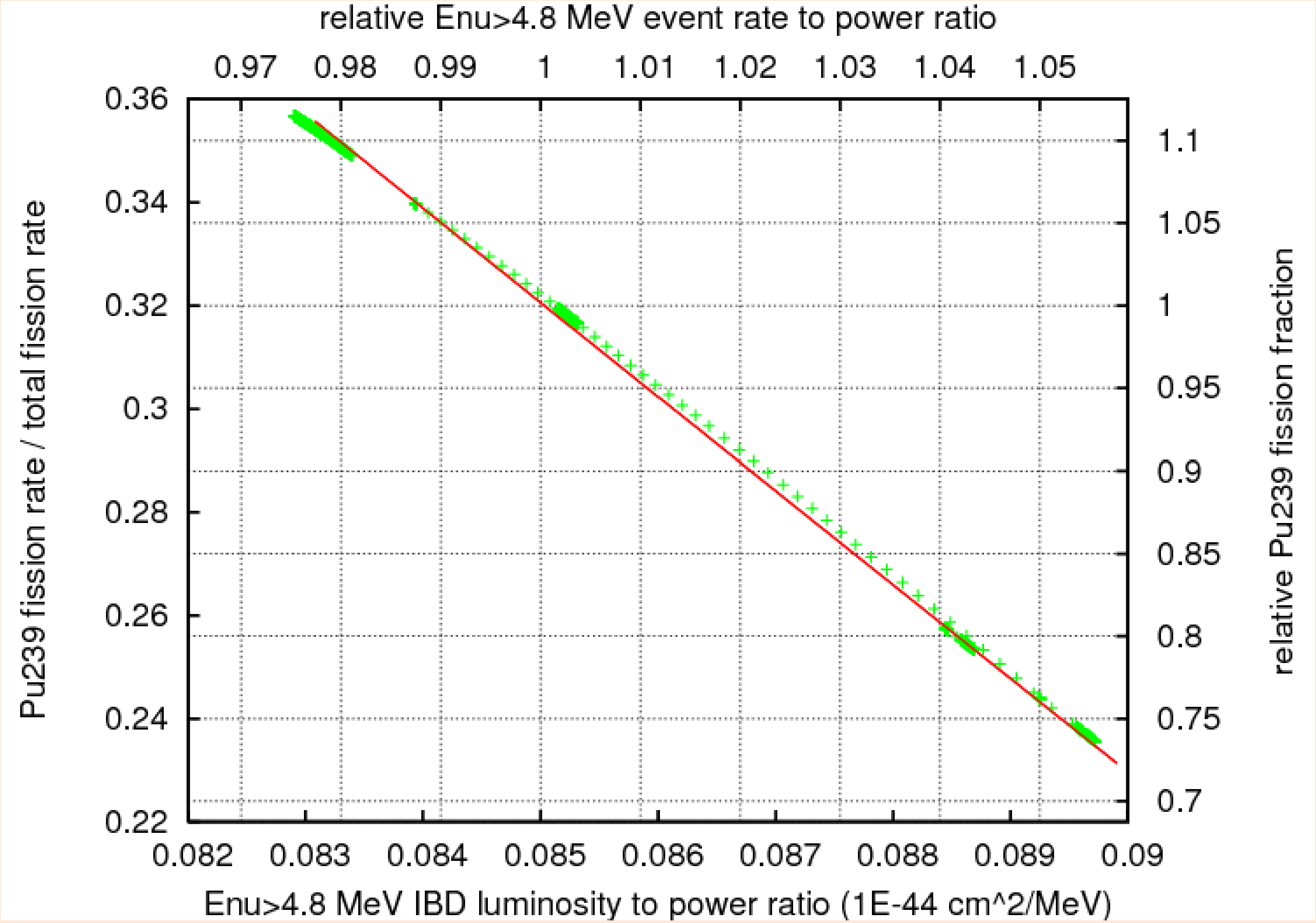
- **MeV/fission:** V. Kopeikin, *et al.*, Phys. Atom. Nucl. 67 (2004) 1892-1899; Yad. Fiz. 67 (2004) 1916-1922; hep-ph/0410100, October 2004.
- **Neutrino spectra:** 235U : K. Schreckenbach *et al.*, Phys. Lett. B 160, 325 (1985); 239,241Pu : A. A. Hahn *et al.*, Phys. Lett. B 218, 365 (1989); 238U : P. Vogel *et al.*, Phys. Rev. C 24, 1543 (1981).
- Sample reactor data provided to Tohoku University by Japanese reactor operators and approved for use in public talks by special agreement. (Next two plots.)



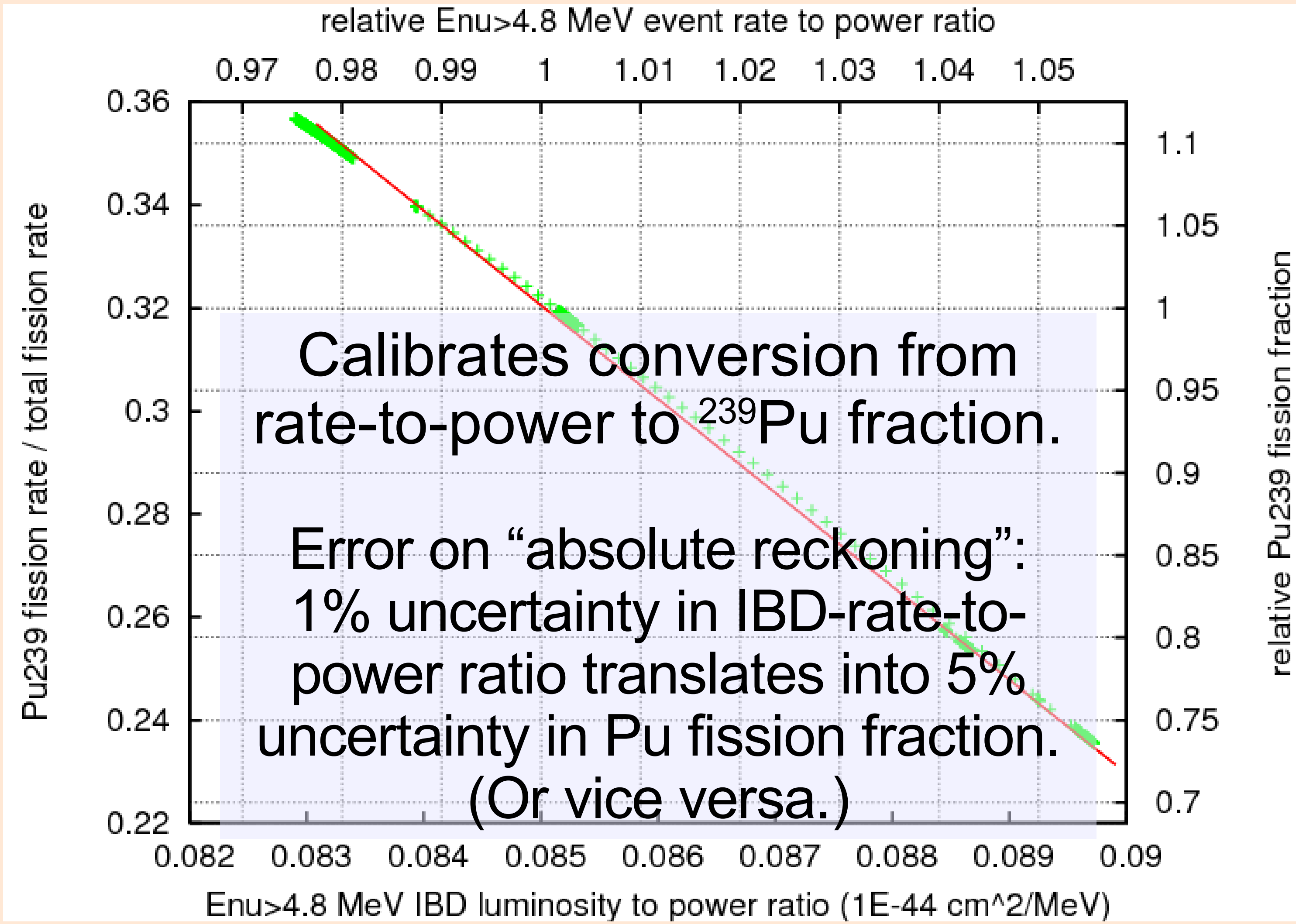
A typical 1.3GWe class BWR in Japan
Data provided according to the special agreement between
Tohoku Univ. and a Japanese nuclear power reactor operator.
Thermal power (MW)
Burnup (GWd/t)
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
0
2
4
6
8
10
12
14
16
18
20
Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
2002
2003
Date


A typical 1.3GWe class BWR in Japan
Data provided according to the special agreement between
Tohoku Univ. and a Japanese nuclear power reactor operator.
U235
Pu239
U238
Pu241
Fission rate (10^19 fission/s)
0
1
2
3
4
5
6
7
8
Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
2002
2003
Date

^{239}Pu fission vs. rate-to-power ratio



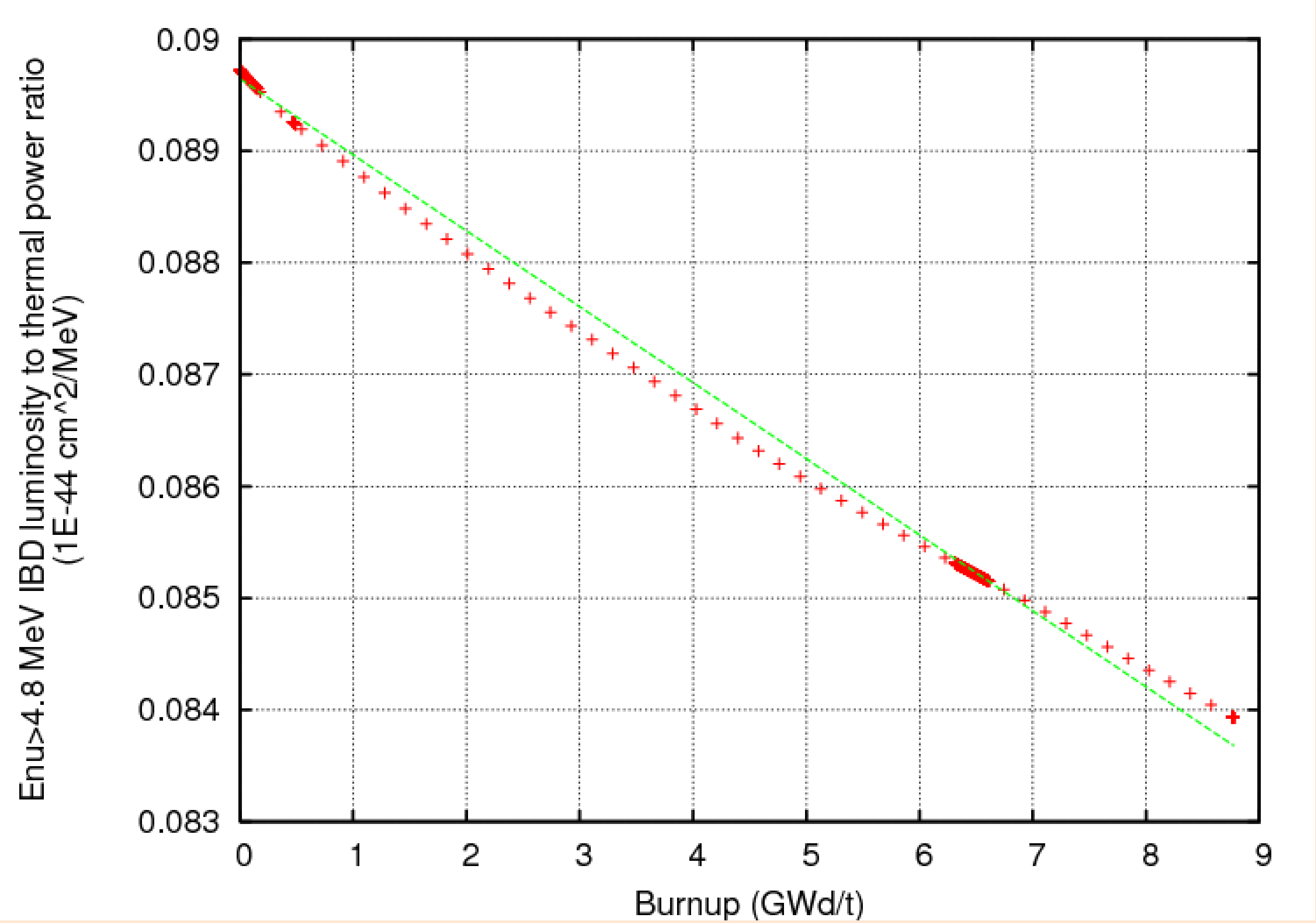
^{239}Pu fission vs. rate-to-power ratio



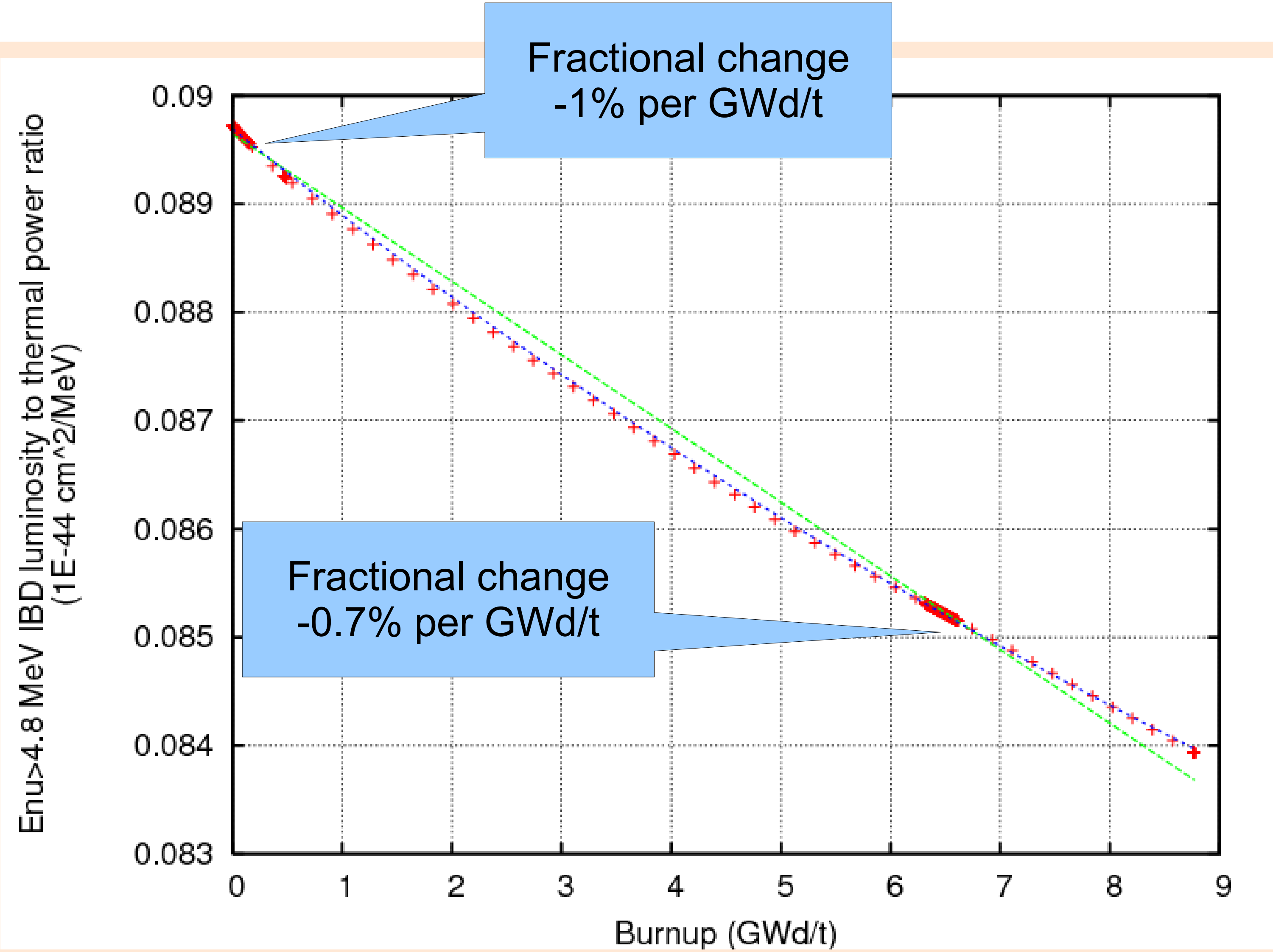
Calibrates conversion from rate-to-power to ^{239}Pu fraction.

Error on “absolute reckoning”: 1% uncertainty in IBD-rate-to-power ratio translates into 5% uncertainty in Pu fission fraction. (Or vice versa.)

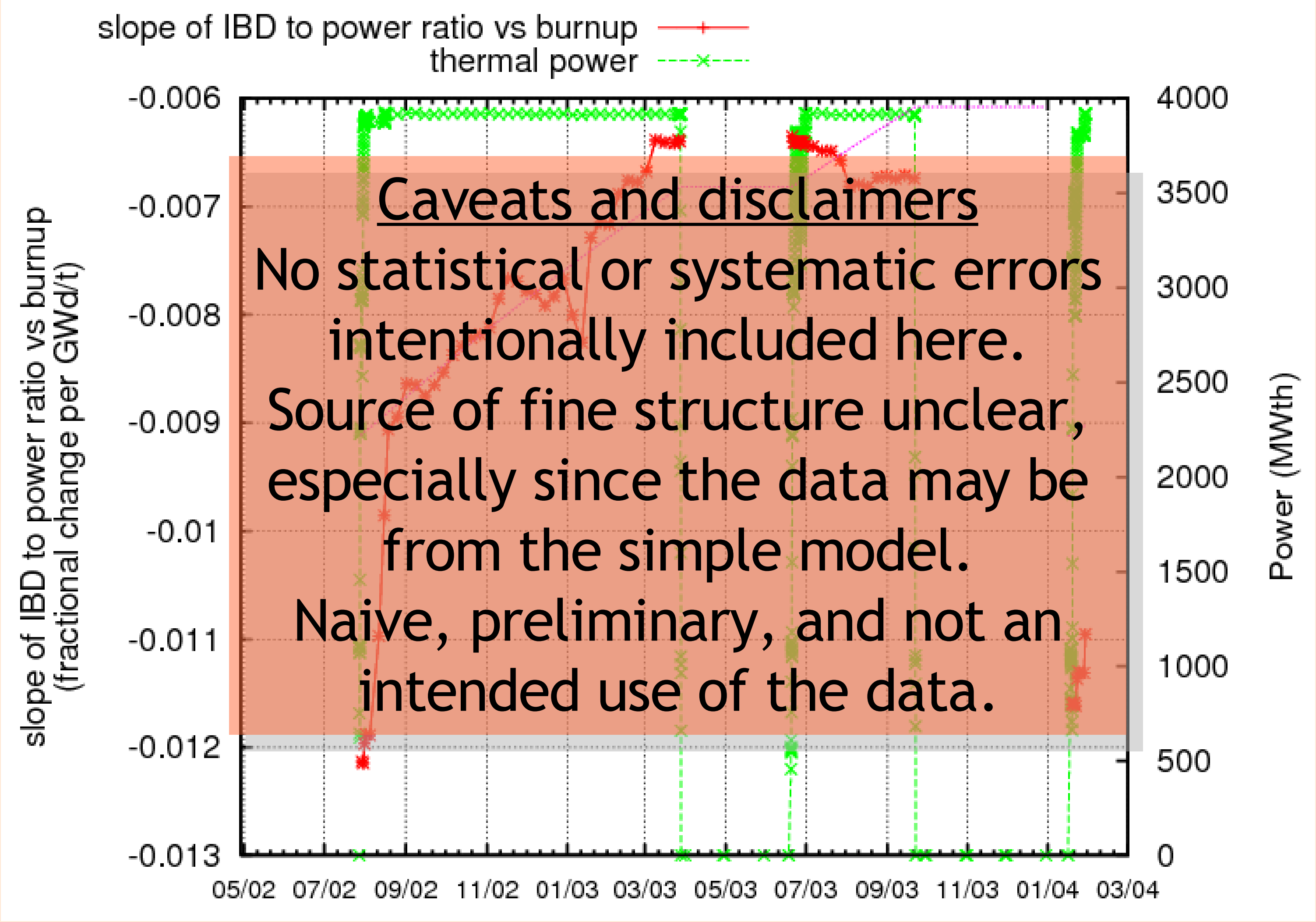
Rate-to-power ratio vs burnup



Rate-to-power ratio vs burnup



“Rate to power vs. burnup slope”
(A plot too far?)



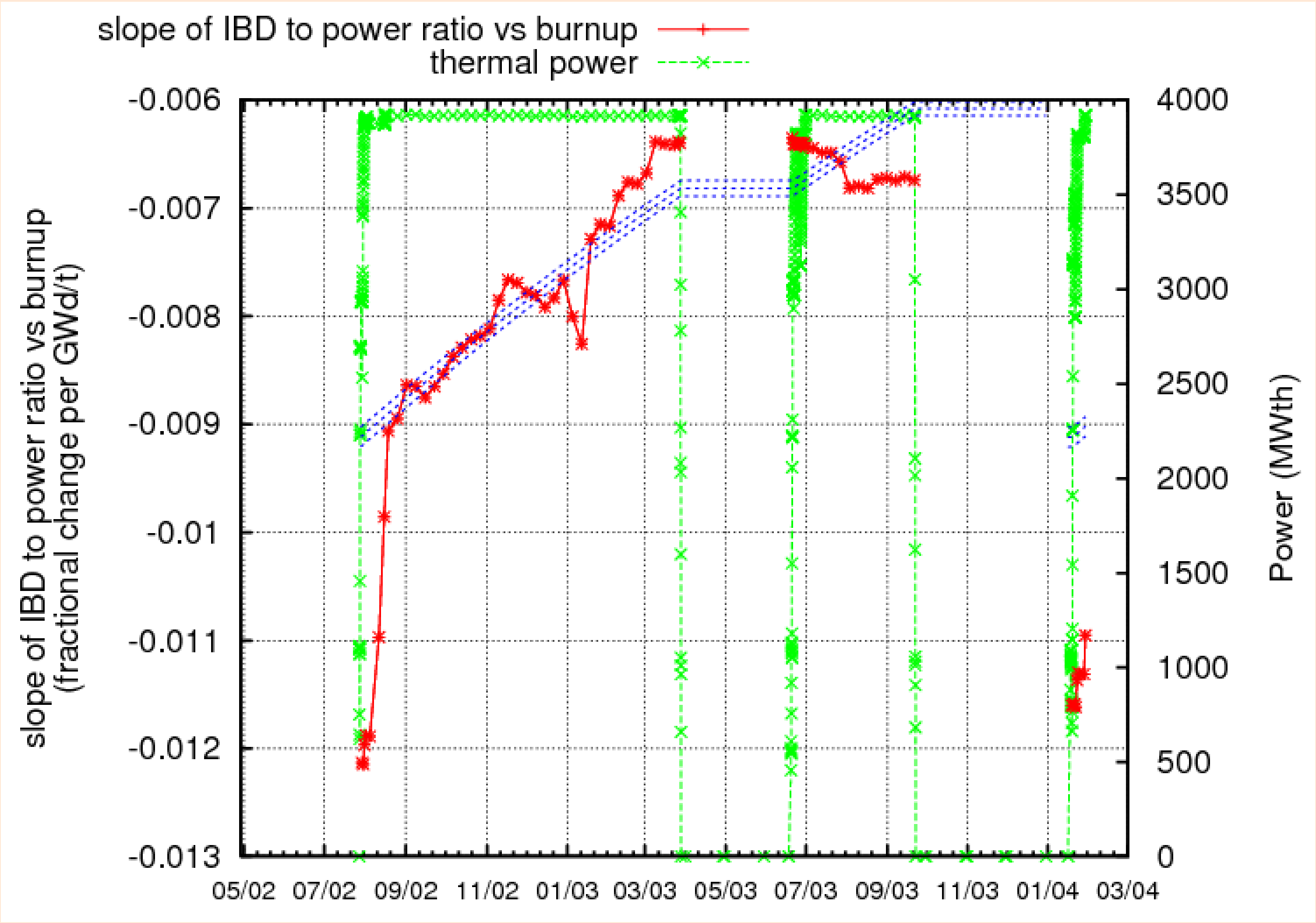
<u>Caveats and disclaimers</u>

No statistical or systematic errors intentionally included here.

Source of fine structure unclear, especially since the data may be from the simple model.

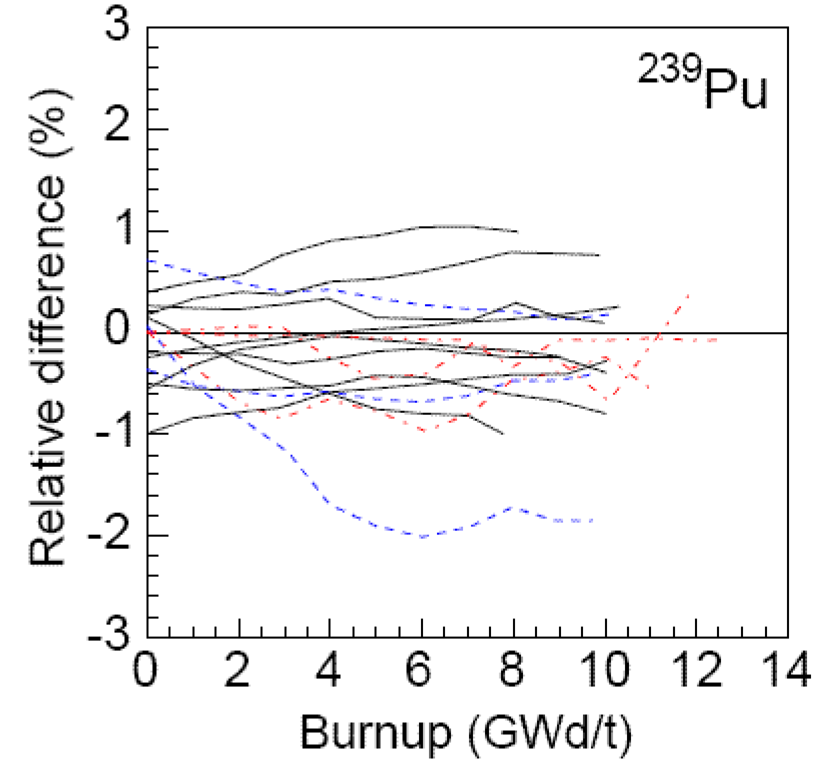
Naive, preliminary, and not an intended use of the data.

"Rate to power vs. burnup slope"
(A plot too far?)



Does the simple model's "rate to power vs. burnup slope" have <1% error?

- If so, would add to ^{239}Pu error less than 1~2% of the ^{239}Pu produced in a single cycle.
- Looking at the plots in the paper, the overall offset seems bigger than curvature.
- Larger week-by-week excursions on the previous page, meaning unclear.



Conclusion

- By special arrangement, reactor operators and neutrino scientists in Japan developed a working simple model to provide neutrino fluxes accurate to 1%, for both PWR and BWRs, with minimal operator data-handling burden.
- Such a model can calibrate the “rate-to-power” to Pu conversion to ~5% accuracy.
- Systematic error contribution to the “rate to power vs. burnup slope” method needs further evaluation.